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Quantum Computation Using Vortices and Majorana Zero Modes of a $p_x + ip_y$ Superfluid of Fermionic Cold Atoms

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We propose to use the recently predicted two-dimensional 'weak-pairing' $p_x + ip_y$ superfluid state of fermionic cold atoms as a platform for topological quantum computation. In the core of a vortex, this state supports a zero-energy Majorana mode, which moves to finite energy in the corresponding topologically trivial 'strong-pairing' state. By braiding vortices in the 'weak-pairing' state, unitary quantum gates can be applied to the Hilbert space of Majorana zero-modes. For read-out of the topological qubits, we propose realistic schemes suitable for atomic superfluids.

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Introduction. Topological quantum computation requires particles that have non-Abelian statistics under interchange and braiding. Under pairwise interchange of particle coordinates, the many-body wave-function of particles following non-Abelian statistics transforms via a unitary transformation in the Hilbert space of a degenerate set of wave-functions. Such particles can arise as the low-energy excitations of a topological phase of matter. One such system, which has been recently discussed in this context [1, 2, 3, 4, 5], is the experimentally observed [6, 7] $\nu = \frac{5}{2}$ fractional quantum Hall state of a two-dimensional (2D) electron gas, where ν is the filling fraction of the electrons. Another promising system is a spin-triplet ($S = 1$), 2D, $p_x + ip_y$ superconductor, in which certain vortex excitations have zero energy Majorana modes[9] in their cores, which endow these vortices with non-Abelian statistics [10, 11, 12]. For a spin-polarized (spinless) $p_x + ip_y$ superconductor, ordinary vortices with vorticity $N = 1$ have such Majorana modes bound at the core [12]. For a $p_x + ip_y$ superconductor with $S_z = 0$, analogous to the A phase of He 3 [13], only the higher-energy vortices with $N = \frac{1}{2}$ – not the lowest energy $N = 1$ vortices – have the Majorana modes. Nevertheless, it is possible to quench the spin-orbit energy, which acts as a confining potential between two $N = \frac{1}{2}$ vortices, by applying a magnetic field [14], and thus to make the Majorana modes potentially realizable in experiments. Based on this appealing idea, and the prospect that strontium ruthenate may be a quasi-2D, $S_z = 0$, $p_x + ip_y$ superconductor [15, 16, 17], it has been recently proposed [18] to use thin films of this material as a system which realize non-Abelian statistics of quasiparticles, associated with these unusual half-quantum (i.e., $N = \frac{1}{2}$) vortices.

Although there is nothing in principle to invalidate such a strategy, practical difficulties may arise due to the lack of quantum coherent motion of vortices in the films. Moreover, since one needs to apply a threshold magnetic field to quench the spin-orbit energy, there will be a relatively high concentration of the half-quantum vortices, thereby rendering independent braiding experimentally challenging. Finally, and most importantly, since the quasiparticles of a superconductor are chargeless, and the Majorana modes are also spinless, there is no simple way to couple to the state of a qubit after a braiding operation has been performed. This makes reading out the state of the qubit difficult. Hence, even though the very realization of non-Abelian statistics through the observation of these vortices is an exciting goal in itself, and for the purposes of topological quantum computation several ideas to overcome the difficulties mentioned above were recently proposed [18], it will really pay to have a $p_x + ip_y$ superfluid system where vortex motion is likely to be coherent, $N = 1$ vortices themselves have non-Abelian statistics so that their concentration can be independently kept low, and a natural read-out scheme exists.

With the recent observation of a p-wave Feshbach resonance in spin-polarized ^{40}K and ^{6}Li atoms in optical traps [19, 20, 21], just such a system – an 'artificially' created $p_x + ip_y$ superfluid of spinless fermions – may now be within experimental reach. Exotic non-Abelian statistics is thus tantalizingly close to fruition in these systems. Since the atoms are in identical spin states, s-wave scattering is Pauli-prohibited and a p-wave resonance dominates, allowing the tunability of the atom-atom interaction in $L = 1$ channel. Recently, it has been theoretically shown [22, 23] that such interactions have the potential to realize various p-wave superfluid states, among them, a $p_x + ip_y$ state in the so-called 'weak-pairing' phase (chemical potential $\mu > 0$) in both three and two dimensions. As a function of the Feshbach resonance detuning, which controls μ, this phase undergoes a topological quantum transition to the strong-pairing phase ($\mu < 0$) [22]. In

2D, the phase with $\mu > 0$ is topologically non-trivial because it supports zero-energy Majorana modes at vortex cores [12], while, as we show below by explicitly constructing the zero-mode wave-function, they disappear in the topologically trivial strong-pairing phase. The weak-pairing phase, then, is suitable for use in the hardware of a quantum computer. Since ordinary vortices themselves exhibit non-Abelian statistics in this case, they can be created at a low density, allowing, in principle, independent braiding. These vortices are also expected to be light due to the high degree of coherence possible in optical traps, and so it is much easier to maintain quantum coherence during the braiding operations. Finally, as we discuss later, since the atoms, unlike the electrons in a superconductor, have internal energy levels, this internal structure can be manipulated to read out the states of the qubits after the braiding operations perform the quantum computation.

The weak and strong pairing phases and the fate of the zero mode. The BCS Hamiltonian for a system of spin-polarized (spinless) fermions in a two-dimensional spin-triplet p-wave superfluid state is given by,

$$H = \int d^2x d^2x' \psi^\dagger(\vec{x})\mathcal{H}(\vec{x},\vec{x}')\psi(\vec{x}'), \quad (1)$$

where $\psi(\vec{x})$ is a two-component column vector, $\psi(\vec{x}) = (c^\dagger(\vec{x}), c(\vec{x}))^{\mathrm{T}}$, and $\mathcal{H}(\vec{x},\vec{x}')$ is the matrix,

$$\mathcal{H}(\vec{x},\vec{x}') = (\frac{-\nabla^2}{2m} - \mu)\delta(\vec{x}-\vec{x}')\sigma_z + \frac{\Delta(\vec{x},\vec{x}')}{2}\sigma^+ - \frac{\Delta^*(\vec{x},\vec{x}')}{2}\sigma^- \quad (2)$$

Here, m is the fermion mass, μ is the chemical potential, and $\Delta(\vec{x},\vec{x}')$ is the gap function. We take $\hbar = k_B = 1$ in this paper. In a uniform $p_x + ip_y$ state, the gap function takes the form in momentum space, $\Delta(\vec{p}) = \frac{\Delta_0}{p_F}(p_x + ip_y)$. For $\mu < 0$, the fall-off of the pair relative wave function $g(\vec{x}_1 - \vec{x}_2)$ is exponential (pairs are tightly bound), whereas, for $\mu > 0$, it is algebraic [12]. Following Ref. [12], we identify the system to be in the strong-pairing phase for $\mu < 0$ and in the weak-pairing phase for $\mu > 0$, the two phases separated by a topological phase transition [24]. The weak-pairing phase supports zero-energy Majorana fermions at the vortex cores. By explicitly constructing the bound state wave function using the Bogoliubov - de Gennes (BdG) equations, below we show that, for $\mu > 0$, the zero-mode quasiparticle is self-hermitian (Majorana), and, as μ is tuned to negative values, these modes disappear in the topologically trivial strong-pairing phase. For $p_x + ip_y$ wave superconductors, analogous BdG equations have been discussed in Refs. 25, 26, 27.

To construct the eigenfunction of the Hamiltonian, Eq. 2, for the zero-energy state, if any, in the presence of a vortex, we model the vortex by assuming the gap function to be zero inside a circular area of radius ξ (coherence length). Outside this radius, the gap function takes the form $\Delta(\vec{p}) = \frac{\Delta_0}{p_F}\exp(i\theta/2)(p_x - ip_y)\exp(i\theta/2)$, where the total order parameter phase, θ, rotates by 2π around the vortex with unit vorticity. In polar coordinates (ρ, θ), for $\rho < \xi$, the BdG equations take the form,

$$[-\frac{1}{2m}(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2}) - \mu]\sigma_z\phi(\vec{x}) = 0, \quad (3)$$

where $\phi(\vec{x}) = (u(\vec{x}), v(\vec{x}))^{\mathrm{T}}$. For the zero-energy state, we take the angular momentum operator $l = -i\frac{\partial}{\partial\theta}$ to have eigenvalue zero. The remaining parts of Eq. 3 imply just ordinary Bessel equations of order zero [28] for both u and v. Since one of the two independent solutions is divergent at the origin, we find the solution for ϕ,

$$\phi(\rho) = AJ_0(\sqrt{2m\mu}\rho)\zeta, \quad (4)$$

where J_0 is the Bessel function of the first kind of order zero, A is a constant, and ζ is a constant spinor.

For solutions with $\rho > \xi$, we note that the gap operator can be written in polar coordinates as, $-i\frac{\Delta_0}{p_F}\exp(-i\theta/2)(\frac{\partial}{\partial\rho} - \frac{i}{\rho}\frac{\partial}{\partial\theta})\exp(i\theta/2) = -i\frac{\Delta_0}{p_F}(\frac{\partial}{\partial\rho} + \frac{1}{2\rho}) - i\frac{\Delta_0}{p_F}l$. Using this, and for zero angular momentum, the BdG equations for the zero-energy state, on multiplication by $-2m\sigma_z$, can be written as,

$$[((((\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho}) + 2m\mu) - \frac{2m\Delta_0}{p_F}(\frac{\partial}{\partial\rho} + \frac{1}{2\rho})\sigma_y)]\phi(\rho) = 0. \quad (5)$$

The solutions to this equation which are well-behaved at $\rho \to \infty$ can be written as $\phi(\rho) = \chi(\rho)\exp(-\frac{\Delta_0}{v_F}\rho)(1, -i)^{\mathrm{T}}$, where $\chi(\rho)$ satisfies

$$[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + (2m\mu - \frac{\Delta_0^2}{v_F^2})]\chi(\rho) = 0, \quad (6)$$

where $v_F = \frac{p_F}{m}$. This is again Bessel equation of order zero. Since both solutions are well-behaved at infinity (they are asymptotically sinusoidal), the general solution for $\phi(\rho)$ can therefore be written as,

$$\phi(\rho) = [BJ_0(\kappa\rho) + CY_0(\kappa\rho)]\exp(i\frac{\pi}{4} - \frac{\Delta_0}{v_F}\rho)(1, -i)^{\mathrm{T}}, \quad (7)$$

where Y_0 is the Bessel function of the second kind of order zero, $\kappa = \sqrt{2m\mu - \Delta_0^2/v_F^2}$, B, C are constants, and the phase factor $e^{i\frac{\pi}{4}}$ is for equal distribution of phase between ϕ and $\phi^\dagger$ (see below). Next, to get a complete solution for the zero-energy state, one needs to match the wave-function and its derivative at $\rho = \xi$, and also normalize the function in all space. These conditions will provide three equations for the three constants A, B, and C, which can then be straightforwardly solved in terms of the known parameters. Once the solution $\phi(\rho)$ and, in turn, $u(\vec{x}), v(\vec{x})$ are known, the quasiparticle operator for the zero-energy state can be written as

$$\gamma_0^\dagger = \int d^2x(u(\vec{x})c^\dagger(\vec{x}) + v(\vec{x})c(\vec{x})) \quad (8)$$

For the overall phase choice $e^{i\frac{\pi}{4}}$ in Eq. 7, one can see from Eq. 8 that $\gamma_0^\dagger = \gamma_0$: the zero energy state is a self-hermitian Majorana state.

To see what happens to the zero-energy state in the strong-pairing phase, this phase can be accessed, in the spirit of Ref. 12, by staying within the mean-field BCS theory and taking $\mu < 0$. Ref. 12 argued that the edge of a vortex could be viewed as a wall separating vacuum with μ large and negative inside the core from the condensate outside. Therefore, in the weak-pairing condensate only, the edge acts like a domain wall between strong (inside the core) and weak-pairing phases, while, in the strong-pairing condensate, nothing interesting should result in the core. By extending the mean-field theory to $\mu < 0$ [25, 29], and replacing μ by $-|\mu|$, Eq. 3 (for zero angular momentum) and Eq. 6 now imply *modified* Bessel equations of order zero for inside and outside the core, respectively. The solution $\phi'(\rho)$ is now given by only one of the two modified Bessel functions of order zero in each case, $I_0(\sqrt{2m|\mu|}\rho)$ for $\rho < \xi$, and $K_0(\kappa'\rho)$ for $\rho > \xi$, since the other one is divergent in the relevant region [28]. Here, $\kappa' = (2m|\mu| + \frac{\Delta_0^2}{v_F^2})^{\frac{1}{2}}$. The corresponding constants multiplying the solutions drop out when one matches the solutions and their derivatives at $\rho = \xi$, and divide one equation by the other. For generic values of the parameters, the resulting equation does not have a solution [29], and therefore, we do not expect a zero-energy state in the strong-pairing phase.

For the sake of completeness, we mention here, that even for a spin-triplet superconductor with $S_z = 0$, in the weak-pairing phase, there are two zero-modes at the core of a vortex with $N = 1$, one for each quasiparticle spin. However, these modes are not Majorana modes, since there is mixing of up and down spin 'c'-operators in the definition of the quasiparticle operator analogous to Eq. 8. Moreover, in the presence of any spin-flip scattering, these degenerate modes will mix and split. This is why one has to consider the exotic $N = \frac{1}{2}$ vortices in these systems in an attempt to realize particles with non-Abelian statistics [18] .

Non-Abelian statistics and unitary operators in the Hilbert space. When the system is in the weak-pairing superfluid phase, a dilute gas of vortices can be created. Suppose there are $2n$ such vortices in the optical trap. Each vortex will have a zero-energy Majorana fermion attached to the core. For $2n$ vortices, there are $2n$ such fermions, which we denote by γ_i, where i counts the vortices. The Majorana fermions can be combined pairwise to create n complex fermionic states, $c_i = \gamma_{2i} + i\gamma_{2i-1}, c_i^\dagger = \gamma_{2i} - i\gamma_{2i-1}$. Each one of these complex fermionic states can be either occupied or unoccupied, giving rise to 2^n-fold degeneracy in the Hilbert space protected by the gap, $\omega_0 \sim \frac{\Delta_0^2}{\epsilon_F}$, where Δ_0 is the amplitude of the pairing gap and ϵ_F is the Fermi energy, to the first excited state in the vortex core. Two states of a representative qubit are identified with the absence ($|0\rangle$) or the presence ($c_i^\dagger|0\rangle$) of a superfluid quasiparticle in the fermionic state constructed from γ_{2i-1} and γ_{2i}. Note that the two states are degenerate and are not directly associated with any particular vortices. It is this non-locality that protects the qubits from decoherence due to the environment, which acts through local operators. For initialization of the qubits, note that creating vortices in pairs from the vacuum will always put each pair in the $|0\rangle$ state at zero temperature (T). At finite T, there is always a non-zero probability that a fermionic quasiparicle will end up on a vortex pair. Since we can read these non-destructively (see below), we can correct or discard the $|1\rangle$'s. We now briefly describe the physics [10] behind the unitary transformations in this space, induced by braiding of the vortices around one another. These unitary transformations can be fruitfully utilized for quantum computation.

Using the property that γ's carry odd charge with respect to the gauge field of a vortex with unit vorticity, that is, $\gamma \to -\gamma$ for a phase change of 2π, it follows that, upon interchange of two neighboring vortices 1 and 2, $\gamma_1 \to \gamma_2$, but $\gamma_2 \to -\gamma_1$ [10]. The unitary operator in the two-dimensional Hilbert space that enforces this transformation is given by,

$$T_1 = \exp(\frac{\pi}{4}\gamma_2\gamma_1) = \exp(i\frac{\pi}{4}(2c^\dagger c - 1)), \tag{9}$$

where, $c = \gamma_1 + i\gamma_2$. This operator can be written as a 2×2 matrix in the space of states spanned by $|0\rangle$ and $c^\dagger|0\rangle$. Likewise, for four vortices $\gamma_1, \gamma_2, \gamma_3$ and γ_4, the unitary braiding operators can be written as 4×4 matrices in the space spanned by the basis states $|0\rangle, c_1^\dagger|0\rangle, c_2^\dagger|0\rangle$ and $c_1^\dagger c_2^\dagger|0\rangle$, where $c_1^\dagger = \gamma_1 + i\gamma_2$, and $c_2^\dagger = \gamma_3 + i\gamma_4$. In the case of $2n$ vortices, the braiding operators are $2^n \times 2^n$-dimensional matrices: they form a matrix representation of the braid group in two dimensions. Upon braiding of two vortices, an initial state, which is now a 2^n-dimensional vector in the space of degenerate states, is multiplied by these matrices and gets transformed to another vector in this space. It is these unitary transformations that can be utilized to build unitary quantum gates, and this is the essence of topological quantum computation. Note that T needs to be kept lower than ω_0. For Feshbach resonance-superfluids, which are near the BCS-BEC transition (recall that it is actually a transition between the weak- and strong-pairing phases, unlike in the s-wave case, in which it is merely a crossover), $\Delta_0 \sim \epsilon_F$, and so T should be kept much lower than ϵ_F, which is realizable in these systems [30]. For this method to succeed, it is imperative that the vortices can be braided around one another like independent particles, which requires a low density of vortices, and their movement be quantum coherent, both of which are achievable in optical traps. Below, we show that the atoms in optical traps also offer a natural strategy for determining the state of

a qubit after a computation has been performed.

Reading out the states of the qubits. A central question in the above scheme is how to determine the state of a qubit after a computation has been performed. The two states of the qubit, as we described above, are distinguished by the presence or absence of a superfluid quasiparticle at the complex fermionic state when two vortices are fused together. However, since these are quasiparticles of a superfluid, they are chargeless. Moreover, since the Majorana fermions are spinless, the complex fermion, which is a linear combination of two Majorana fermions, is also spinless. Therefore, one does not expect an excess of charge or spin due to the presence of a quasiparticle at the core of the composite vortex. One may look for subtle differences in charge or energy *distribution* at the core due to the presence or absence of a quasiparticle, but these may be experimentally difficult to achieve.

A different approach, suitable for atomic (or molecular) superfluids only, is to use the internal energy levels of the atoms themselves. The basic point is that, if there is an unpaired atom at the core of the composite vortex (the qubit is in the state $c^{\dagger}|0\rangle$), photons from a laser can be absorbed to excite the atom to an appropriately chosen excited level. If there is no quasiparticle there (the qubit is in the state $|0\rangle$), there will be no absorption at this frequency. Note that, during this process, one might end up exciting Cooper pairs from outside the core as well. However, to excite an atom bound in a Cooper pair with another atom in an identical internal state, one first needs to break the pair, costing an energy $2\Delta_0$. Thus, from this process, photons can only be absorbed at a frequency shifted by $2\Delta_0$. Since the typical spontaneous emission rate, $\sim \mathcal{O}(2\pi \times 1 \text{ MHz})$, in such a detection process is much larger than $\Delta_0 \sim 2\pi \times 11$ KHz [30], this method can be applied only through intermediate states which induce a much larger energy splitting between paired and unpaired atoms. Here we illustrate this reading out scheme using ^{40}K atoms, although the technique is applicable to other species as well.

Suppose the atoms in the superfluid are in the $4^2S_{1/2}$ hyperfine ground state $|i\rangle \equiv |F = 9/2, m_F = -7/2\rangle$ in the case of p-wave resonance [19]. To determine whether there is an unpaired atom inside a composite vortex, a two-photon Raman pulse is applied that transfers the unpaired atom to another hyperfine state $|j\rangle \equiv |F = 7/2, m_F = -5/2\rangle$. The frequency difference between the two Raman lasers is adjusted to be resonant with the hyperfine splitting between states $|i\rangle$ and $|j\rangle$ for the unpaired atom, but has a $2\Delta_0$ detuning for paired atoms due to the energy cost to break a pair. The lasers have maximal intensities located at the core of the vortex and their beam waist width $w \approx 1.5\ \mu m$ is much smaller than the typical distance ($\geq 10\mu m$) between vortices [31], allowing individual access to the qubits. The Rabi frequency of the Raman pulse is chosen to have a Gaussian shape $\Omega = \Omega_0 \exp\left(-\omega_0^2 t^2\right)$ $(-t_f \leq t \leq t_f)$ to reduce the impact on paired atoms [32]. For a set of parameters $\Delta_0 = 2\pi \times 11$ KHz, $\omega_0 = \Delta_0/2$, and $\Omega_0 = 1.77\omega_0$, $t_f = 5/\omega_0$ [32], we find that the unpaired atom is completely transferred from state $|i\rangle$ to $|j\rangle$ by the Raman pulse, while the probability for the paired atoms to be excited to state $|j\rangle$ is about 6×10^{-6} and may therefore be neglected.

To obtain a cycling transition necessary for the detection of the unpaired atom, π Raman pulses are applied to transfer the unpaired atom to state $|k\rangle \equiv |F = 9/2, m_F = 9/2\rangle$. Because of large Zeeman splitting between different magnetic sublevels, these Raman pulses may be performed in a short period (no longer than $100\mu s$). A focused σ^+-polarized detection laser resonant with the cycling transition $|k\rangle \rightarrow |l\rangle \equiv \left|5^2P_{3/2} : F = 11/2, m_F = 11/2\right\rangle$ is then applied to detect atoms at state $|k\rangle$. Here we choose the $4S \rightarrow 5P$ instead of $4S \rightarrow 4P$ transition for the detection laser to obtain smaller diffraction limit as well as smaller spontaneous decay rate. In the experiment [19], the magnetic field $B \approx 200$ G for the p-wave Feshbach resonance, which yields an effective detuning $\delta \approx 2\pi \times 170$ MHz for the paired atoms at state $|i\rangle$. The ratio between the number of the spontaneously emitted photons by paired and unpaired atoms is estimated to be $(\Gamma/2\delta)^2 \approx 1.2 \times 10^{-5}$, where $\Gamma \approx 2\pi \times 1.2$ MHz is the decay rate for the excited state $|l\rangle$. Therefore, in the detection process, the impact on paired atoms may be neglected, and resonant fluorescence is observed if and only if initially there is one unpaired atom inside the vortex at state $|i\rangle$. For this read-out scheme to succeed, the temperature needs to be kept as low as possible, so that there are no other unpaired atoms in the bulk around the vortex cores. However, such thermally excited quasiparticles are expected to occur near the trap edges, and so for lasers sufficiently focused on the vortices near the center of the trap, one should be able to significantly suppress the unwanted signals. Finally, we mention that the resonant detection laser may be replaced with a resonant multiphoton ionization process of the unpaired atom, yielding a single ionized electron that can be detected with essentially unit efficiency [33]. Similar analysis as above also yields negligible impact on paired atoms. The advantage of the multiphoton ionization is that the detection may be done in parallel for all unpaired atoms in the sense that the electrons can be imaged on a channel plate, where the detection or no detection of the electrons is a parallel readout of all the qubits. However, such process is destructive and the ionized atoms cannot be reused.

In conclusion, we have proposed to use the two-dimensional, spin-polarized, $p_x + ip_y$ atomic resonance-superfluid in the weak-pairing phase, potentially realizable in optical traps, in a suitable hardware for topological quantum computation. We have given a realistic read-out scheme for the topological qubits, a major hurdle in this field, using the internal states of the con-

stituent atoms.

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